Ch. 2 Test in class part   Name: Key

Double check your solutions! Use Algebraic Notation AND Show All of Your Work. No Assistance or Collaboration! You may not leave to use the restroom. Calculator part.

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1. (5 points) Convert 61°42′21″ to a decimal in degrees. Round your answer to two decimal places
   1. $\boxed{61.71^\circ}$

$61^\circ + 42' + 21''$

$$= 61^\circ + 42' \cdot \left(\frac{1^\circ}{60'}\right) + 21'' \cdot \left(\frac{1^\circ}{3600''}\right)$$

$$\doteq 61^\circ + 0.7^\circ + 0.0058^\circ = 61.71^\circ$$

2. (5 points) Convert 61.24° to $D^\circ M' S''$ form. Round your answer to the nearest second.
   2. $\boxed{61^\circ\ 14'\ 24''}$

$$61.24^\circ = 61^\circ + 0.24^\circ \cdot \left(\frac{60'}{1^\circ}\right)$$

$$= 61^\circ + 14.4'$$

$$= 61^\circ + 14' + 0.4'\left(\frac{60''}{1'}\right)$$

$$= 61^\circ\ 14' + 24''$$

3. (5 points) A carnival has a Ferris wheel whose radius is 27 feet. You measure the time it takes for one revolution to be 63 seconds. What is the linear speed (in feet per second) of the Ferris wheel? What is the angular speed in radians per second?

Given: $r = 27$ ft

$\omega = \frac{1\text{rev}}{63\text{sec}}$

3. ________

$$\omega = \frac{1\text{rev}}{63\text{sec}} \cdot \frac{2\pi \text{ rads}}{1\text{rev}} =$$

$$\boxed{\omega = \frac{2\pi \text{ rad}}{63 \text{ sec}} \text{ or } 0.1 \text{ rads/sec}}$$

Then

$$V = r\omega = \cancel{27}^{3} \text{ ft} \cdot \frac{2\pi}{\cancel{63}_{7} \text{ sec}}$$

$$\boxed{V = \frac{6\pi}{7} \text{ ft/sec} \quad \text{or} \quad 2.7 \text{ ft/sec}}$$

4. (5 points) A water sprinkler sprays water over a distance of 30 feet while rotating through and angle of 135°. What area of lawn receives water?

4. ________

Given $r = 30$ ft, $\theta = 135° \cdot \left(\frac{\pi}{180°}\right) = \frac{3\pi}{4}$

$$A = \frac{1}{2} r^2 \theta = \frac{1}{2}(30 \text{ ft})^2 \cdot \frac{3\pi}{4} = \frac{2700\pi \text{ ft}^2}{8} = \frac{675\pi}{2} \text{ ft}^2$$

$$\boxed{\begin{array}{l} A = 337.5\pi \text{ ft}^2 \\ \text{or} \\ A \doteq 1061 \text{ ft}^2 \end{array}}$$

5. (2 points) Find $\sin(21°)$. Round to the nearest tenth. 5. 0.4

6. (2 points) Find $\tan(3)$. Round to the nearest tenth. 6. -0.1

7. (2 points) Find $\csc(17)$. Round to the nearest tenth. 7. -1.0

8. (5 points) Find the length of the arc of a circle of radius 2 meters subtended by the central angle $\theta = 120°$.

8. $\boxed{\frac{4\pi}{3} m \quad \text{or} \quad 4.2 m}$

Given

$r = 2m$ and $\theta = 120° = \frac{2\pi}{3}$

Find s.

$$s = r\theta = (2m)\left(\frac{2\pi}{3}\right) = \frac{4\pi}{3} m$$

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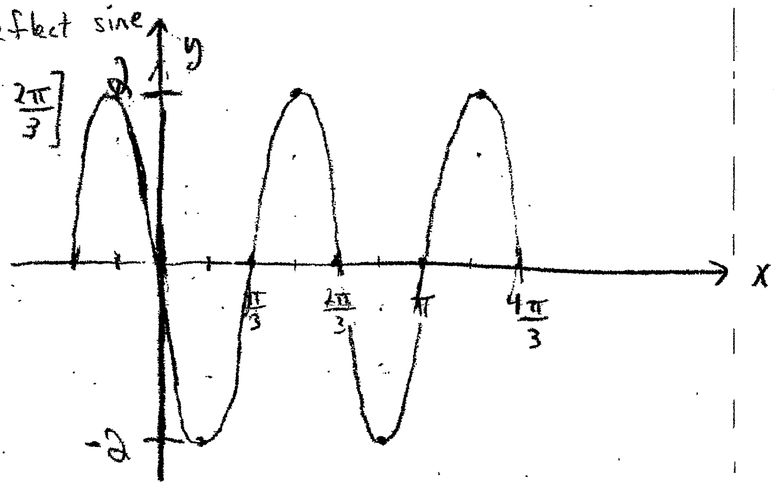
9. (5 points) Graph two periods of the function $y = -2\sin(3x)$

$[A = -2] \Rightarrow$ reflect sine

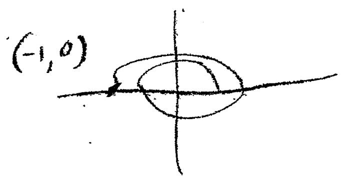
$[\omega = 3] \Rightarrow [Pd = \frac{2\pi}{3}]$



10. (5 points) What is the domain and range of $y = -2\sin(3x)$

10. dom = $\mathbb{R}$, rng = $[-2, 2]$

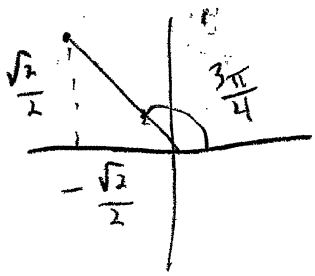
11. (5 points) Find $\tan(3\pi) = \frac{y}{x} = \frac{0}{-1} = 0$



11. 0

12. (5 points) Find $\cos\left(\frac{11\pi}{4}\right)$

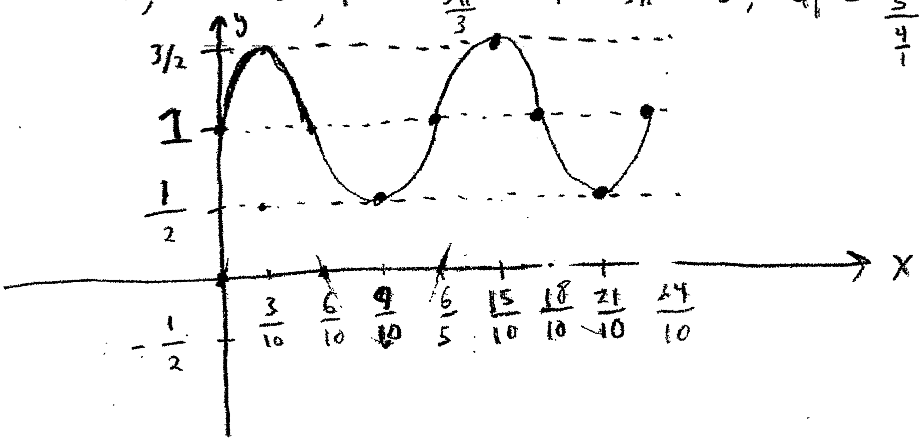
12. $\boxed{-\frac{\sqrt{2}}{2}}$



$$\cos\left(\frac{11\pi}{4}\right) = \cos\left(\frac{3\pi}{4} + \frac{8\pi}{4}\right)$$
$$= \cos\left(\frac{3\pi}{4} + 2\pi\right)$$
$$= \cos\left(\frac{3\pi}{4}\right) = -\frac{\sqrt{2}}{2}$$

13. (5 points) Graph two periods of the function $y = \frac{1}{2}\sin\left(\frac{5\pi}{3}x\right) + 1$.

$A = \frac{1}{2}$, $\omega = \frac{5\pi}{3}$, $pd = \frac{2\pi}{\frac{5\pi}{3}} = \frac{2\pi}{1} \cdot \frac{3}{5\pi} = \frac{6}{5}$, $Qp = \frac{\frac{6}{5}}{\frac{4}{1}} = \frac{6}{5} \cdot \frac{1}{4} = \frac{3}{10}$



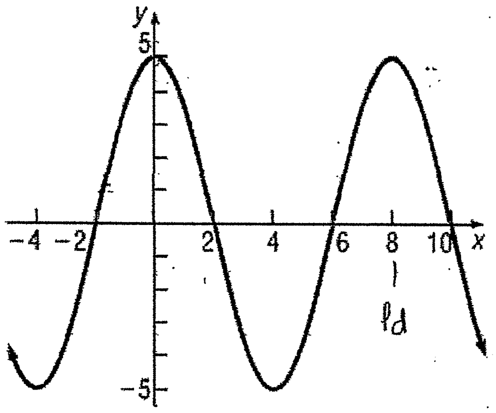
14. (5 points) What is the domain and range of $y = \frac{1}{2}\sin\left(\frac{5\pi}{3}x\right) + 1$

dom = $\mathbb{R}$

14. rng = $[1/2, 3/2]$

15. (5 points) Find an equation for the graph below.

15. $\boxed{y = 5\cos\left(\frac{\pi}{4}x\right)}$



cosine graph

$A = 5$

Pd $= 8$

$\frac{2\pi}{\omega} = 8$

$2\pi = 8\omega$

$\frac{2\pi}{8} = \omega$

16. (5 points) Find the exact value of each of the remaining trig functions of θ. Assume

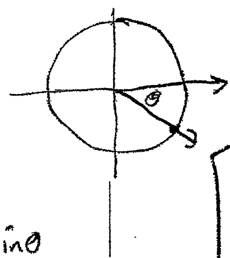
$$\sec(\theta) = 2 \qquad \text{and} \qquad \sin(\theta) < 0$$

$x < 0$, $y \geq 0$	$x > 0$, $y > 0$
$x < 0$, $y < 0$	$x > 0$, $y < 0$

$[\sec\theta > 0] \Rightarrow [\theta \in Q1 \text{ or } Q4]$

$[\sin\theta < 0] \Rightarrow [\theta \in Q3 \text{ or } Q4]$

So θ must be a Q4 angle



16. $\boxed{\begin{array}{ll} \cos\theta = \frac{1}{2} & \sin\theta = -\frac{\sqrt{3}}{2} \\ \sec\theta = 2 & \csc\theta = -\frac{2\sqrt{3}}{3} \\ \tan\theta = -\sqrt{3} & \cot\theta = -\frac{\sqrt{3}}{3} \end{array}}$

$\cos^2\theta + \sin^2\theta = 1$

$\left(\frac{1}{2}\right)^2 + \sin^2\theta = 1$

$\sin^2\theta = 1 - \frac{1}{4}$

$\sin^2\theta = \frac{3}{4}$

$\sin\theta = -\sqrt{\frac{3}{4}} = -\frac{\sqrt{3}}{2}$

Then $\csc\theta = -\frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

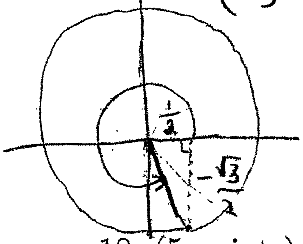
$= \frac{-\frac{\sqrt{3}}{2}}{\frac{1}{2}}$

$= -\sqrt{3}$ and $\cot\theta = -\frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$

17. (5 points) Find $\sec\left(\frac{11\pi}{3}\right)$

17. $\boxed{2}$

$$= \sec\left(\frac{5\pi}{3}+\frac{6\pi}{3}\right) = \sec\left(\frac{5\pi}{3}+2\pi\right) = \sec\left(\frac{5\pi}{3}\right)$$

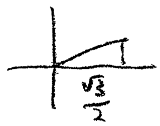


$$= \frac{1}{\cos\left(\frac{5\pi}{3}\right)} = \frac{1}{\frac{1}{2}} = 2$$

18. (5 points) Find $\sec 390°$

18. $\boxed{\frac{2\sqrt{3}}{3}}$

$$\sec(390°) = \sec(30°+360°)$$



$$= \sec(30°)$$

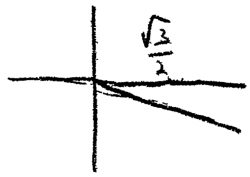
$$= \frac{1}{\frac{\sqrt{3}}{2}} = 1 \cdot \frac{2}{\sqrt{3}} = \frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$$

19. (5 points) Find $\cos 690°$

19. $\boxed{\frac{\sqrt{3}}{2}}$

$$\cos(690°) = \cos(330°+360°)$$

$$\begin{array}{r} 690 \\ -360 \\ \hline 330 \end{array}$$



$$= \cos(330°)$$

$$= \frac{\sqrt{3}}{2}$$

20. (5 points) Find the exact value of $\frac{\sin(-20°)}{\cos(380°)} + \tan(200°)$

20. $\boxed{0}$

$$= \frac{-\sin(20°)}{\cos(20°)} + \tan(20°)$$

Since $\sin(-\theta) = -\sin\theta$, $\cos(\theta+360°) = \cos\theta$, $\tan(\theta+180°) = \tan\theta$

$$= -\tan 20° + \tan 20°$$

Since $\tan\theta = \frac{\sin\theta}{\cos\theta}$

$$= 0$$

Ch. 2 Test Takehome Part Name: Key

Graph at least two periods of each function given below. Also, find the domain and range for each function.

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2 1. $y = \csc(2x) - 1$

2 2. $y = -2\sec(4x) - 1$

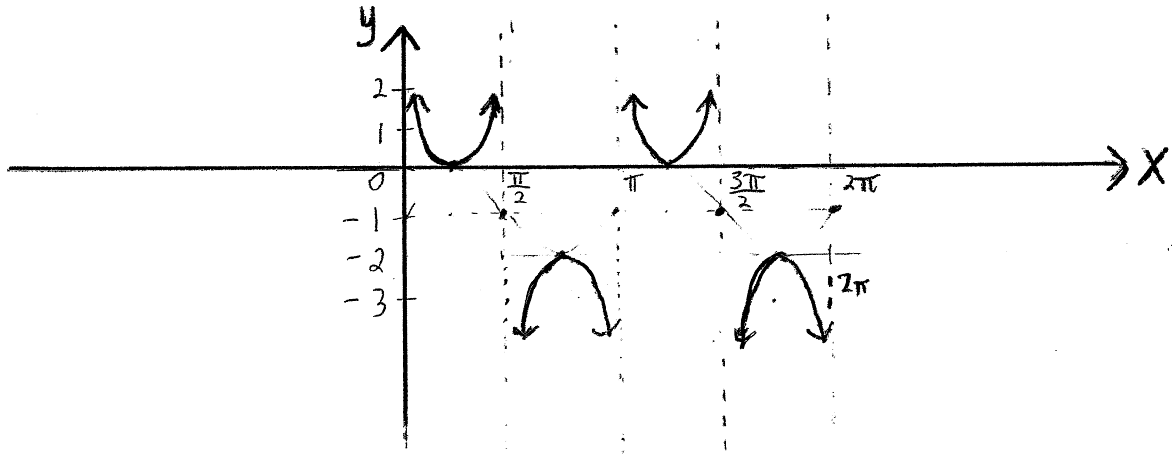
2 3. $y = 3\tan\left(\frac{\pi}{3}x\right)$

2 4. $y = 4\cot\left(\frac{\pi}{6}x\right)$

(21) $y = \csc(2x) - 1$

$A = 1$, $pd = \frac{2\pi}{2} = \pi$, vertical shift -1

$$\boxed{\begin{array}{l} dom = \{x \mid x \neq \frac{k\pi}{2}\} \\ rng = (-\infty, -2] \cup [0, \infty) \end{array}}$$



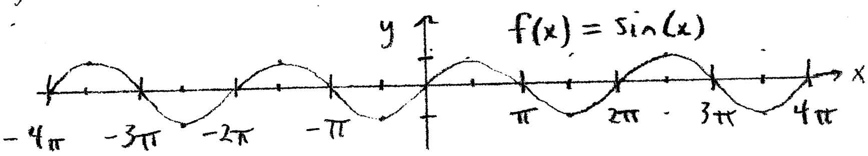
domain: Asymptotes are at $\frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi, \ldots$

Also,

$$y = \csc(2x) - 1$$

$$= \frac{1}{\sin(2x)} - 1$$

We know that our function has vertical asymptotes whenever $y = \sin(2x)$ has x-intercepts. We know that $f(x) = \sin(x)$ has x-intercepts at any integer multiple of π, or at $x = k\pi$, where $k \in \mathbb{Z}$.



To build $y = \sin(2x)$ from $f(x) = \sin(x)$, we replace x with $2x$. So to find x-intercepts for $y = \sin(2x)$, replace the x in $x = k\pi$ with $2x$, then solve for x.

$$2x = k\pi$$

$$\frac{2x}{2} = \frac{k\pi}{2} \Rightarrow \left[\text{The set of x-intercepts for } y = \sin(2x) = \{x \mid x = \frac{k\pi}{2}\} \right]$$

$$\Rightarrow \left[\text{The set of vertical asymptotes for } y = \csc(2x) - 1 = \{x \mid x = \frac{k\pi}{2}\} \right]$$

$$\Rightarrow \left[dom(\csc(2x) - 1) = \{x \mid x \neq \frac{k\pi}{2}\} \right]$$

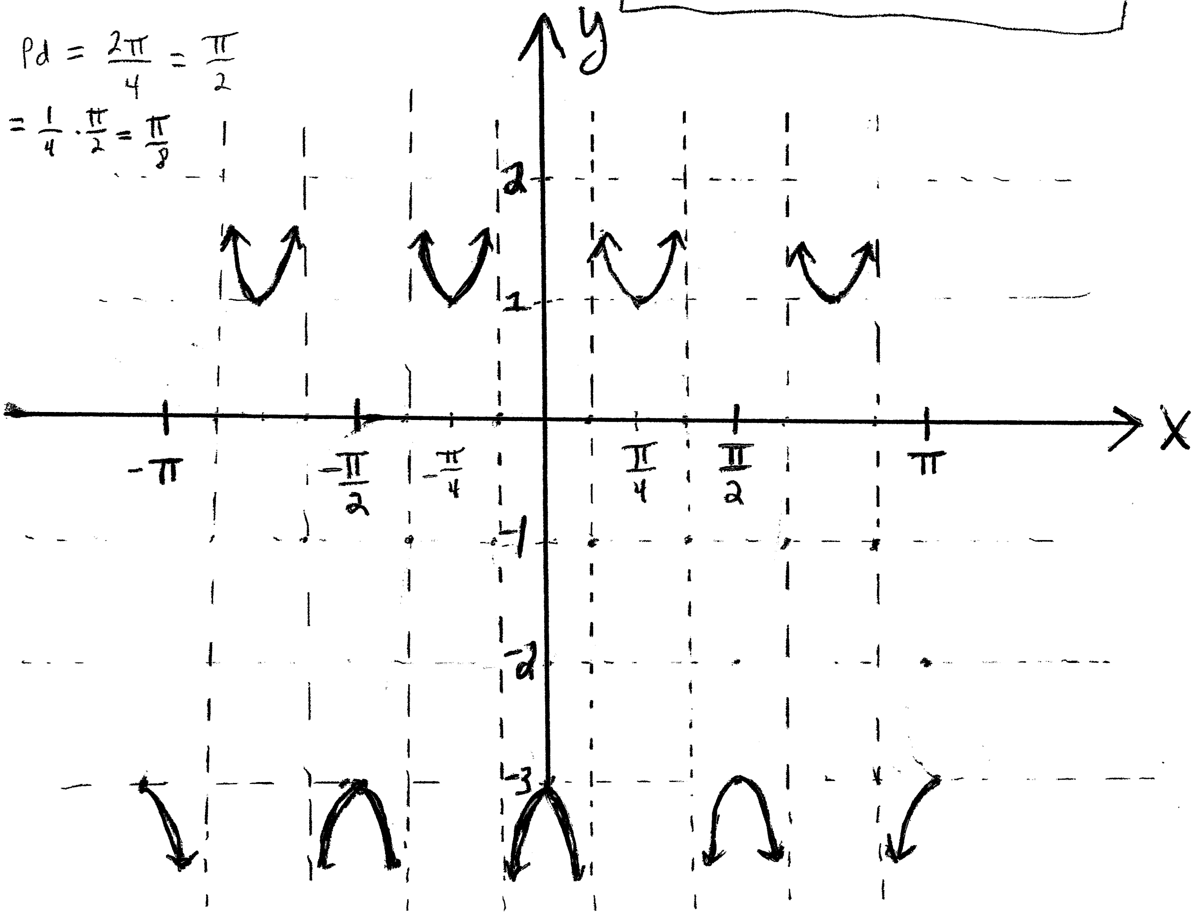
(22) $y = -2\sec(4x) - 1$

$A = -2$ reflect cosine

$$\boxed{\begin{aligned} dom &= \left\{x \mid x \neq \frac{(1+2k)\pi}{8}\right\} \\ rng &= (-\infty, -3] \cup [1, \infty) \end{aligned}}$$

$Pd = \frac{2\pi}{4} = \frac{\pi}{2}$

$QP = \frac{1}{4} \cdot \frac{\pi}{2} = \frac{\pi}{8}$

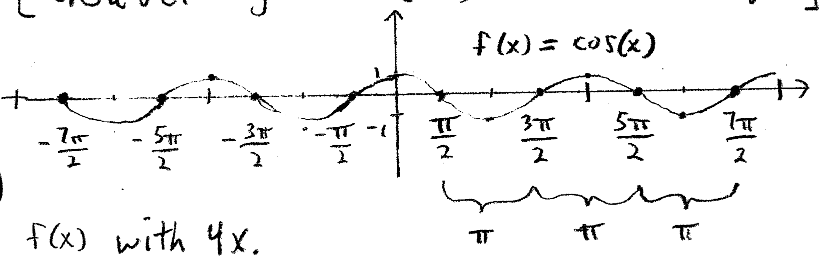


domain: Asymtotes at $x = \frac{\pi}{8}, \frac{3\pi}{8}, \frac{5\pi}{8}, \ldots$

Also, $y = -2\sec(4x) - 1$, or

$y = -2 \cdot \frac{1}{\cos(4x)} - 1 \Rightarrow$ [our function has vertical asymptotes wherever $y = \cos(4x)$ has x-intercepts.]

we know that $f(x) = \cos(x)$ has x-intercepts at $x = \frac{\pi}{2} + k\pi$, where k is any integer. To build $y = \cos(4x)$ from $f(x) = \cos(x)$, replace the x in f(x) with 4x.



To find the x-intercepts of $y = \cos(4x)$, we replace the x in $x = \frac{\pi}{2} + k\pi$ with 4x, and solve for x. Then $4x = \frac{\pi}{2} + k\pi$. Multiply both sides by $\frac{1}{4}$ to solve for x,

$x = \frac{1}{4}\left(\frac{\pi}{2} + k\pi\right)$ or $\boxed{x = \frac{\pi}{8} + \frac{k\pi}{4}}$ or $x = \frac{\pi}{8} + \frac{k\pi}{4} \cdot \frac{2}{2}$ or $\boxed{x = \frac{\pi + 2\pi k}{8} = \frac{(1+2k)\pi}{8}}$

(23) $y = 3\tan\left(\frac{\pi}{3}x\right)$

$A = 3$

$Pd = \frac{\pi}{\omega} = \frac{\pi}{\frac{\pi}{3}} = \frac{\pi}{1} \cdot \frac{3}{\pi} = 3$

domain, range & vertical asymptotes: $rng = (-\infty, \infty)$

VA's $\frac{\pi}{3}x = \frac{\pi}{2} + k\pi$ multiply both sides by $\frac{3}{\pi}$ to solve for x.

$$\frac{3}{\pi} \cdot \frac{\pi}{3} \cdot x = \frac{3}{\pi}\left(\frac{\pi}{2} + k\pi\right)$$

$$1x = \frac{3}{\pi} \cdot \frac{\pi}{2} + \frac{3}{\pi} \cdot k\pi$$

$$x = \frac{3}{2} + 3k \Rightarrow \left[VA's = \{x \mid x = \tfrac{3}{2} + 3k,\ k \in \mathbb{Z}\}\right]$$

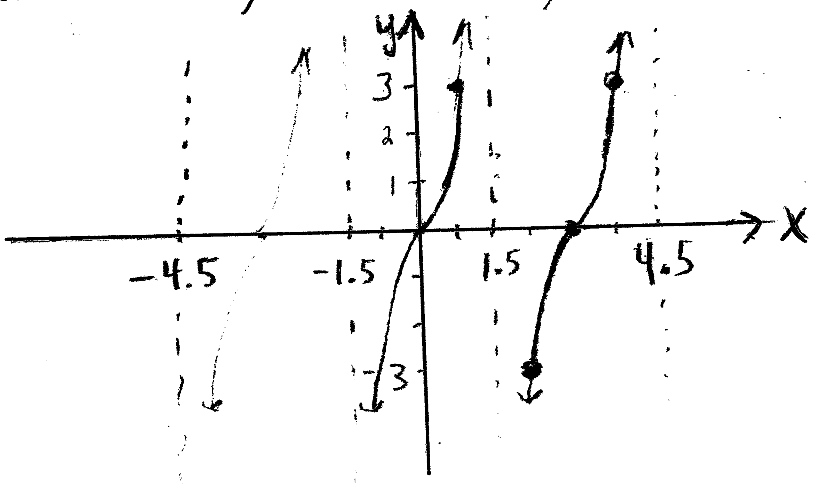
$$\Rightarrow \left[dom = \{x \mid x \neq \tfrac{3}{2} + 3k,\ k \in \mathbb{Z}\}\right]$$

Find 3 VA's near the origin for a 2-period graph:

When $k = 0$, $x = \frac{3}{2} + 3 \cdot 0$, or $x = 1.5$ is a VA

When $k = 1$, $x = \frac{3}{2} + 3 \cdot 1$, or $x = 4.5$ is a VA

When $k = -1$, $x = 3/2 + 3 \cdot (-1)$, or $x = -1.5$ is a VA



$$\boxed{\begin{array}{l} dom = \{x \mid x \neq \tfrac{3}{2} + 3k\} \\ rng = \mathbb{R} \end{array}}$$

(24) $y = 4\cot\left(\frac{\pi}{6}x\right)$

$A = 1$

$Pd = \frac{\pi}{\omega} = \frac{\pi}{\frac{\pi}{6}} = \frac{\pi}{1} \cdot \frac{6}{\pi} = 6$

domain, range and vertical asymptotes: $rng = (-\infty, \infty)$

VA's $\quad \frac{\pi}{6}x = K\pi \quad$ multiply both sides by $\frac{6}{\pi}$ to solve for x.

$$\frac{6}{\pi} \cdot \frac{\pi}{6} \cdot x = \frac{6}{\pi} \cdot K\pi$$

$$1x = 6K \Rightarrow \left[VA's = \{x \mid x = 6k, \text{ where } K \in \mathbb{Z}\} \right]$$

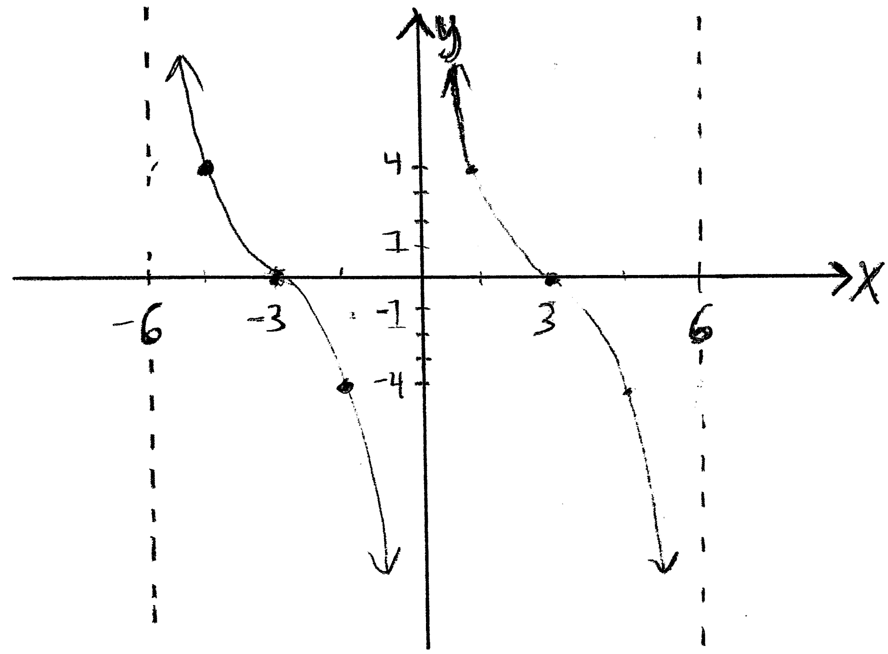
$$\Rightarrow \left[dom = \{x \mid x \neq 6k, \ k \in \mathbb{Z}\} \right]$$

Find 3 VA's near the origin for a 2pd graph:

When $K = 0$, $x = 6 \cdot 0$ or $x = 0$ is a VA. (the y-axis is a vertical asymptote

When $K = 1$, $x = 6$ is a VA.

When $k = -1$, $x = -6$ is a VA.



$$\boxed{\begin{array}{l} dom = \{x \mid x \neq 6K\} \\ rng = \mathbb{R} \end{array}}$$